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An SO(10) $\times$ S_4 Scenario for Naturally Degenerate Neutrinos[1]

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Abstract

The simplest scenario for the three known light neutrinos that fits the solar and atmospheric neutrino deficit and a mixed dark matter (MDM) picture of the universe requires them to be highly degenerate with $m_\nu \sim 1$ - 2 eV. We propose an SO(10) grand unified model with an S_4-horizontal symmetry that leads naturally to such a scenario. An explicit numerical analysis of the quark and lepton sector of the model shows that it can lead to desired mass differences to fit all data only for the small angle non-adiabatic MSW solution to the solar neutrino puzzle.

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In the two recent papers, it has been pointed out by Caldwell and one of the authors (R.N.M.)[1] that if the existing data on solar[2] and atmospheric neutrino[3] deficit is confirmed and if the presently popular mixed dark matter (MDM)[4] model of the universe requiring a few eV neutrino as its hot component is taken seriously, then the simplest three-light neutrino (ν_e, ν_μ, ν_τ) mass matrix that fits all data has the following form:

$$M = \begin{pmatrix} m+\delta_1 s_1^2 & -\delta_1 c_1 c_2 s_1 & -\delta_1 c_1 s_1 s_2 \\ -\delta_1 c_1 c_2 s_1 & m+\delta_1 c_1^2 c_2^2+\delta_2 s_2^2 & (\delta_1-\delta_2)s_2 c_2 \\ -\delta_1 c_1 s_1 s_2 & (\delta_1-\delta_2)s_2 c_2 & m+\delta_1 s_2^2+\delta_2 c_2^2 \end{pmatrix}, \tag{1}$$

where m=2 eV, s$_1 \sim .05$, s$_2 \sim .38$, $\delta_1 \sim 1.5\times 10^{-6}$ eV and $\delta_1 \sim .2$ to .002 eV. Here we have assumed that the solar neutrino puzzle is solved by the small angle MSW (Mikheyev-Smirnov-Wolfenstein)[5] solution[6]. Note the high degree of degeneracy among the three neutrino species. An immediate implication of this is that if the neutrinos are Majorana particles , the neutrinoless double beta decay would be measurable in the current generation experiments involving ^{76}Ge[7] and ^{130}Te[8] thereby providing a test of the degenerate neutrino hypothesis.

It is then perhaps not premature to search for gauge models which can generate this highly degenerate neutrino spectrum[9] in a technically natural manner. Such degeneracy is suggestive of a horizontal symmetry, which will contain all three neutrinos in one irreducible representation. This symmetry however must be broken in the charged lepton sector. Since the see-saw mechanism[10] generally connects the neutrino masses with the charged fermion masses , the horizontal symmetry breaking in the charged fermion sector will cause mass differences between the neutrinos. Such mass differences are of course required and from equation (1), we see that these must be very tiny. A simple scaling argument then says that the see-saw scale must be in the range of 10^{12} GeV or so. Such mass scales have their natural place in grand unified theories and we will consider SO(10) as the flavor grand unifying theory in order to understand the required neutrino spectrum.

Turning now to the horizontal symmetry, an obvious choice is to consider it to be $SU(2)_H$ as has been done in several recent papers[1, 9]. In this letter, we consider a somewhat more economical group based on the permutation group S_4 as our horizontal symmetry. We will assume that the symmetry is softly broken so that there is no domain wall problem. We find that in this model, the fermion sector is completely specified by fifteen arbitrary parameters, twelve of which are fixed by the charged fermion sector (i.e. by six quark masses, three CKM angles and three charged lepton masses). The structure of the neutrino mass matrix is then such that

only the small angle MSW solution to the solar neutrino problem,

Before presenting the details of the model, let us first review the basic strategy given in Ref. [1]. It is well-known that when the conventional see-saw mechanism for neutrino-masses[10] is implemented in gauge models such as SO(10) or the left-right symmetric models, it gets modified to the following form[11]

$$\begin{pmatrix} f v_L & m_{\upsilon^D} \\ m^T_{\upsilon^D} & f v_R \end{pmatrix}, \tag{2}$$

where

$$v_L = \lambda \frac{v^2_{wk} v_R}{M^2_P}; \tag{3}$$

v_R is the scale of SU$(2)_R$-breaking and M_P is breaking scale of parity. Therefore, unless special care is taken to break parity symmetry at a scale higher than the SU$(2)_R$ or U$(1)_{B-L}$, $v_L \sim \lambda v^2_{wk}/v_R$ (since $v_R \sim M_P$). The light neutrino masses are then given by:

$$m_\nu \simeq f v_L - \frac{m_{\nu^D} f^{-1} m^T_{\nu^D}}{v_R}. \tag{4}$$

Recall that the conventional see-saw formula omits the first term (which, as just mentioned, is justified only under special circumstances) leading to an approximate quadratic scaling relation between neutrino and up-quark mass (or in some instances charged lepton masses). We will however keep both the terms in the present discussion. Now notice that if due to some symmetry reasons, $f_{ab} = f_0 \delta_{ab}$, then a degenerate neutrino spectrum emerges.

Before discussing the fermion masses, let us briefly review some of the discussions of Ref. [1]. We will consider the breaking of SO(10) $\rightarrow$ SU$(2)_L$ $\times$ SU$(2)_R$ $\times$ SU$(4)_C$ $\times$ P (denoted by G_{224P}) by means of a $\{\mathbf{54}\}$-dim. Higgs multiplet. This symmetry is subsequently broken down to the standard model by a $\{\mathbf{126}\}$-dim. Higgs multiplet. Detailed two-loop analysis of the mass scales in this model[12] leads to $v_R \sim 10^{13.6}$ GeV. So that for $f_0\lambda \sim 1/2$, we get $f_0 v_L \sim 1$ eV, as desired.

Let us now turn to the implication of S$_4$-symmetry for the charged fermion and neutrino masses, which is the main contribution of this paper. S$_4$-symmetry has been used before in the discussion of charged fermion masses at the electroweak

level[13]. It has irreducible representations with dimension $\{\mathbf{3}\}$, $\{\mathbf{3}'\}$, $\{\mathbf{2}\}$, $\{\mathbf{1}\}$, and $\{\mathbf{1}'\}$. Our assignment of fermions and Higgs multiplets to irreducible representations of S_4 are shown in Table I.

The S_4-invariant Yukawa coupling can be written symbolically as

$$
\begin{aligned}
L_Y &= \frac{1}{\sqrt{3}}(\Psi_1\Psi_1 + \Psi_2\Psi_2 + \Psi_3\Psi_3)(h_0 H_0 + f\bar{\Delta}_0) \\
&+ \frac{1}{\sqrt{2}}\left[(\Psi_3\Psi_2 + \Psi_2\Psi_3)(h_2 H_1 + f_2\bar{\Delta}_1) + (\Psi_3\Psi_3 + \Psi_2\Psi_2 - 2\Psi_1\Psi_1)(h_2 H_2 + f_2\bar{\Delta}_2)\right] \\
&+ \frac{f_3}{\sqrt{2}}\left[(\Psi_1\Psi_3 + \Psi_3\Psi_1)H_3 + (\Psi_2\Psi_1 + \Psi_1\Psi_2)H_4 + (\Psi_3\Psi_3 - \Psi_2\Psi_2)H_5\right] \\
&+ H.\ c. \qquad (5)
\end{aligned}
$$

We then assume that the S_4-symmetry is softly broken by the masses of H_i ($i = 0, 1, ..., 5$) and Δ_i ($i = 0, 1, 2$), so that their vacuum expectation values are arbitrary. We also assume a softly broken $U(1)_{PQ}$ symmetry so that the complex $\{\mathbf{10}\}$'s have only one coupling to the fermions. The H_i's therefore have vev's given by κ_i^u and κ_i^d. Turning now to the Δ's, we choose only the (mass)^2 of Δ_0 negative and large so that the $(\mathbf{1}, \mathbf{3}, \overline{\mathbf{10}})$ submultiplet of it (the numbers denote representation under the group G_{224P}) acquire a vev v_R that breaks G_{224P} down to the standard model. The remaining two Δ's ($\Delta_{1,2}$) have large (M_U) positive masses so that their (**2,2,15**) components acquire induced vev's of order of the electroweak scale without any fine tuning[14] due to the presence of $S_4\times$ $U(1)_{PQ}$ invariant terms such as $\Delta_0\overline{\Delta}_0\Delta_i H_i$ in their potential (where i=1,2). A term of the form $\Delta_0\overline{\Delta}_0\Delta_0 H_0$ also induces non-zero vev's to the (**2,2,15**) submultiplets of Δ_0. We denote these Δ-vev's by v_i^u, v_i^d ($i = 0, 1, 2$). Note that the Δ_0 coupling in Eq. (5) leads after symmetry breaking to the degenerate neutrino masses.

The charged fermion and Dirac-neutrino mass matrices can now be written as follows.

$$
\begin{aligned}
M_{u,ab} &= m_{u,ab}^{(10)} + f_0 v^u \delta_{ab} + m_{u,ab}^{(126)} \\
M_{d,ab} &= m_{d,ab}^{(10)} + f_0 v^d \delta_{ab} + m_{d,ab}^{(126)} \\
M_{l,ab} &= m_{d,ab}^{(10)} - 3f_0 v^d \delta_{ab} - 3m_{d,ab}^{(126)} \\
M_{\nu^D,ab} &= m_{u,ab}^{(10)} - 3f_0 v^u \delta_{ab} - 3m_{u,ab}^{(126)} \\
M_{\nu,ab} &= f_0 v_L \delta_{ab} - \left(M_{\nu^D}^2\right)_{ab}/(f_0 v_R), \qquad (6)
\end{aligned}
$$

where

$$m_{u,ab}^{(10)} = \begin{pmatrix} a_0 - 2a_2 & a_4 & a_3 \\ a_4 & a_0 + a_2 - a_5 & a_1 \\ a_3 & a_1 & a_0 + a_2 + a_5 \end{pmatrix}, \quad (7)$$

$$m_{u,ab}^{(126)} + f_0 v^u \delta_{ab} = \begin{pmatrix} d_0 - 2d_2 & 0 & 0 \\ 0 & d_0 + d_2 & d_1 \\ 0 & d_1 & d_0 + d_2 \end{pmatrix}, \quad (8)$$

where a_i's and d_i's are products of type $h\kappa^u$ and fv^u respectively. Similar matrices can be written down for $m_{d,ab}^{(10)}$ and $m_{d,ab}^{(126)}$ by replacing a_i's by b_i's , κ^u by κ^d and d_i's by e_i's. This way of writing makes it clear that there are a total of 18 parameters. However, we can choose a basis in which the up-quark mass matrix is diagonal, thereby getting rid of three parameters and we are left with fifteen parameters. We will now fit these parameters so that the six quark masses, three charged lepton masses and three CKM angles are reproduced. In order to proceed , we first extrapolate these parameters from the electro-weak scale to the scale M_R, where their values are:

$$\begin{array}{lll} m_u = .0013923, & m_c = .36322 & m_t = 75.900, \\ m_d = .0024297, & m_s = .047775 & m_b = 1.38975, \\ s_{12} = \pm.2200, & s_{13} = .00624 & s_{23} = .05200, \\ m_e = .0004896, & m_\mu = .10080 & m_\tau = 1.71264. \end{array}$$

We have chosen all masses to be positive. We do the fitting as follows. In the basis where M_u is diagonal, we have $M_d = U_{CKM} M_d^{(diag)} U_{CKM}^\dagger$. The first choice leads to six constraints on the original (before choosing the up-quark basis) nine parameters leaving three arbitrary. We choose them to be a_0, a_1, and a_2. Similarly the M_d equation leaves three arbitrary parameters chosen to be b_0, b_1, and b_2. The b_0, b_1, and b_2 are then determined by three equations that relate them to TrM_l, TrM_l^2 and TrM_l^3 which of course are easily expressed in terms of the m_e, m_μ, and m_τ . The matrix M_l is then completely determined. Diagonalizing it as usual i.e. $M_l = U_E M_l^{(diag)} U_E^\dagger$, where $M_l^{(diag)} = \mathrm{Diag}(m_e,\ m_\mu,\ m_\tau)$, the three angles that parameterize V_E are predicted. In particular, note that, in the basis in which M_u is diagonal, $\nu_e \nu_\mu$ mixing arises predominantly from the (e, μ, τ) sector due to the fact that M_{ν^D} is block diagonal and fitting m_e, m_μ, and m_τ leads a prediction for

$\theta_{\nu_e\nu_\mu}$. We find that for negative s_{12}, two sets of solutions for the mixing matrix V_E correspond to the correct charged lepton masses. Only one of them leads to the desired ν_e-ν_μ mixing angle and we choose this one ; the parameters a_0, a_1, and a_2 not fixed by the up-quark sector, are then varied to obtain the required mass difference squares between m_{ν_e}, m_{ν_μ}, and m_{ν_τ} to fit the data.

Let us now briefly discuss the dependence of our solutions on the three parameters a_0,a_1 and a_2. Requiring $\Delta m^2_{\nu_\mu-\nu_\tau} \simeq .1eV^2$ implies that, $a_0 + a_2 \simeq 38GeV$; if we further assume that the non-degenarate part of the contribution to the neutrino masses have a hierarchical structure between the first and the second generation leading to $\Delta m^2_{\nu_e-\nu_\mu} \simeq 10^{-6}eV^2$, this leads to $a_0 \simeq 2a_2 \simeq 25.36GeV$ and $a_1 \leq 2$. In figure 1, we have shown the dependence of the ν_e- ν_μ mass difference on a_1 for the above choices of a_2 and a_0 and we see that the desired range comes only for $a_1 \leq 2$. We give below the detailed solution for the neutrino masses and mixings for this case. (We have written $m_{\nu_i} = m_0 + m'_{\nu_i}$, where m_0 is the direct v_L contribution.)

$$(m'_{\nu_e}, m'_{\nu_\mu}, m'_{\nu_\tau}) = \frac{1}{f v_R}(-0.0000174465, -0.129248, -5759.27)\mathrm{GeV}^2,$$

and

$$V^l \;=\; \begin{pmatrix} -.9982 & .05733 & .01476 \\ .05884 & .9334 & .3541 \\ -.006523 & -.3544 & .9351 \end{pmatrix}. \tag{9}$$

Note that, for $v_R \simeq 10^{13.6}$ GeV and $f \sim 3$, this predicts $|m^2_{\nu_\mu} - m^2_{\nu_e}| \sim 4\times 10^{-6}$ eV^2 for $m_0 = 2$ eV, $|m^2_{\nu_\tau} - m^2_{\nu_\mu}| \sim .2$ eV^2, which are in the range required to solve both the solar and atmospheric neutrino deficit for the values of $\theta_{\nu_e\nu_\mu}$ and $\theta_{\nu_\mu\nu_\tau}$ given above. In particular, we wish to note the preference of theory for the small angle MSW solution to the solar neutrino problem. This is quite interesting in view of the recent conclusions[15] that the large angle MSW solutions have a discouragingly large χ^2-fit.

We wish to remark that our solutions do not depend on the choice of sign for the up quark masses. We have checked that we lose the desired values of masses and mixings if we choose m_s to be negative for both signs of the Cabibbo angle.

In conclusion, we have constructed an SO(10) $\times$ S$_4$ model, which leads to a degenerate neutrino mass scenario including correct mixing angles required to fit atmospheric and solar neutrino data as well as the hot dark matter in the universe. A critical test of the model is the observation of a non-zero signal in neutrinoless

double data decay in the current generation of ^{76}Ge and ^{130}Te experiments. Also our model prefers only the small angle MSW solution to the solar neutrino puzzle, a result which is already indicated in recent analyses and will surely be tested once more data accumulates.

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Table Caption

Table I: Transformation property of fermions and Higgs multiplets under S_4-symmetry. The $\{\mathbf{10}\}$-dimensional multiplets are chosen to be complex..

Multiplet		S_4 representation
Fermions		
$\Psi_a, a = 1, 2, 3$		$\{\mathbf{3}\}$
Higgs Bosons		
$\{\mathbf{126}\}$	Δ_0	$\{\mathbf{1}\}$
$\{\mathbf{126}\}$	$\Delta_{1,2}$	$\{\mathbf{2}\}$
$\{\mathbf{10}\}$	H_0	$\{\mathbf{1}\}$
$\{\mathbf{10}\}$	$H_{1,2}$	$\{\mathbf{2}\}$
$\{\mathbf{10}\}$	$H_{3,4,5}$	$\{\mathbf{3}\}$

Table I.

Figure Captions

Fig. 1: m'_{ν_e} and m'_{ν_μ} are plotted as a function of a_1 for $a_0 = 2a_2 = 25.38 GeV$. The scale of the vertical axes is 8.37×10^{-6} eV for $v_R = 10^{13.6}$ GeV and $f = 3$.